Quadratic Functions

- Explain the meaning of the term function, and distinguish a function from a relation that is not a function, through investigation of linear and quadratic relations using a variety of representations (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies.

- Substitute into and evaluate linear and quadratic functions represented using function notation, including functions arising from real-world applications.

- Explain the meanings of the terms domain and range, through investigations using numeric, graphical, and algebraic representations of linear and quadratic functions, and describe the domain and range of a function appropriately.

- Explain any restrictions on the domain and the range of a quadratic function in contexts arising from real-world applications.

- Determine, through investigation using technology, the roles of \( a, h, \) and \( k \) in quadratic functions of the form \( f(x) = a(x - h)^2 + k \), and describe these roles in terms of transformations on the graph of \( f(x) = x^2 \) (i.e., translations; reflections in the \( x \)-axis; vertical stretches and compressions to and from the \( x \)-axis).

- Sketch graphs of \( g(x) = a(x - h)^2 + k \) by applying one or more transformations to the graph of \( f(x) = x^2 \).

- Collect data that can be modelled as a quadratic function, through investigation with and without technology, from primary sources, using a variety of tools, or from secondary sources, and graph the data.

- Determine, through investigation using a variety of strategies, the equation of the quadratic function that best models a suitable data set graphed on a scatter plot, and compare this equation to the equation of a curve of best fit generated with technology.

- Solve problems arising from real-world applications, given the algebraic representation of a quadratic function.
A satellite dish is constructed and positioned in such a way that the signals it receives are directed to focus at a point for transmission to your television. The cross-section of these satellite dishes takes the shape of a parabola. You will also find parabolic shapes in architecture, bridges, and paths of projectiles. The graphs for quadratic functions that model area, revenue, and many other real-life situations are often parabolic in shape. In this chapter, you will connect the graphs and equations of quadratic functions in real-world applications.

**Vocabulary**
- axis of symmetry
- domain
- first differences
- function
- integer
- mapping diagram
- parabola
- quadratic function
- range
- real number
- relation
- second differences
- transformation
- translation
- vertex
- vertical compression
- vertical line test
- vertical stretch
Substitute into Equations

1. Find the value of $y$ when $x = 0$.
   a) $y = 4x - 5$
   b) $y = -5x^2 + 10x + 2$
   c) $y^2 = x$
   d) $7x - 3y = 21$

2. Find the value of $y$.
   a) $y = 4x - 5$ when $x = 3$
   b) $y = x^2 + 4x - 5$ when $x = -2$
   c) $3x - 5y = 8$ when $x = 6$
   d) $x^2 + y^2 = 9$ when $x = -3$

Create Tables to Draw Graphs

3. Copy and complete each table.
   a) $y = -3x + 7$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
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<tr>
<td>-1</td>
<td></td>
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<tr>
<td>0</td>
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<tr>
<td>1</td>
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<td>2</td>
<td></td>
</tr>
</tbody>
</table>

   b) $y = x^2 - 2x + 1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
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<td>-1</td>
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<td>2</td>
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</tbody>
</table>

4. Use Technology Refer to question 3.
   a) Graph each relation on grid paper.
   b) Use a graphing calculator to check your graphs in part a).

Calculate First and Second Differences

5. Copy and complete each table.
   a) $y = 2x + 1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>First Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
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<tr>
<td>-2</td>
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<td>-1</td>
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<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b) $y = 3x^2 - 6x$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>First Differences</th>
<th>Second Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
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<tr>
<td>3</td>
<td></td>
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</tbody>
</table>
Transformations

6. Find the image of the point (2, 3) after each transformation.
   a) a translation of 3 units to the right
   b) a translation of 8 units down
   c) a reflection in the x-axis

7. Copy the graph for $y = x^2$ on grid paper.
   Draw the following on the same set of axes.
   a) the reflection of the graph in the x-axis
   b) the image of the graph after a translation of 4 units to the left and 5 units down

Calculate Area and Volume

8. A piece of cardboard measures 40 cm by 40 cm. An open-topped box is to be constructed by removing a square with side length 8 cm from each corner and folding up the edges.
   a) Find the surface area of the box.
   b) Find the volume of the box.

Chapter Problem

Lina is the promoter of a concert. She has researched the effect of ticket prices on ticket sales. Results show that 10 000 people will attend the concert if the price is $20 per ticket. For each dollar increase in price, 1000 fewer people will attend.

If the stadium can hold 15 000 people, will a ticket price of $10 be likely? Justify your answer. What problems can arise if the ticket price is too high or too low?
Identify Functions

A pay cheque from your job often shows more data than just the amount of money you receive. Payroll software accepts the input of your data, performs calculations, and then produces a number that represents your pay. What are the data for the input? Why must there be a single result?

Investigate A

How do you determine if a relation is a function?

A: Relations With Parabolic Graphs

The equations and graphs of two relations are shown. The equations look alike in their forms. However, Relation A represents a function while Relation B does not represent a function.

Function
Relation A: \( y = x^2 \)

Non-Function
Relation B: \( y^2 = x \)

1. Copy and complete each table. You may write more than one \( y \)-value for each \( x \)-value.

Relation A: \( y = x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
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<tr>
<td>0</td>
<td></td>
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<tr>
<td>1</td>
<td></td>
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<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Relation B: \( y^2 = x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
2. In Relation B, when \(x = 4\), \(y = 2\) or \(-2\). Locate the points on the graph. Where are the points relative to each other?

3. This is a **mapping diagram** for Relation B. Draw a mapping diagram for Relation A.

4. Record the number of arrows starting from each \(x\)-value in the mapping diagram for Relation A. Is the result the same as for Relation B?

5. Relation A is considered a function while Relation B is not considered a function. Describe how the two relations differ, • as equations • as graphs • as tables of values • as mapping diagrams

6. What properties must a relation have to be a function?

**B: Relations With Non-Parabolic Graphs**

The equations and graphs of another two relations are shown. Relation C represents a function while Relation D does not represent a function.

**Function**
Relation C: \(x + y = 5\)

**Non-Function**
Relation D: \(x^2 + y^2 = 25\)

1. Hold a ruler vertically and move it slowly from left to right on the graphs of Relations C and D. Observe the number of points that the edge of the ruler intersects each graph. Repeat this process for Relations A and B. What do you notice?
2. Copy and complete each table. You may write more than one y-value for each x-value.

Relation C: $x + y = 5$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
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<tr>
<td>0</td>
<td></td>
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<td>1</td>
<td></td>
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<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Relation D: $x^2 + y^2 = 25$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td></td>
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<td>0</td>
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<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

3. In Relation D, find the y-intercepts. What are their values?

4. This is a mapping diagram for Relation C. Draw a mapping diagram for Relation D. How many arrows do you think will start from $x = 0$?

5. Examine the number of arrows starting from each $x$-value in the two relations C and D. What differences did you find?

C: Summarize

1. Compare the two functions (Relation A and Relation C) to the two non-functions (Relation B and Relation D) in parts A and B of this investigation. Consider the various representations: equation, graph, table of values, and mapping diagram, as well as the vertical line test. A table similar to this one may be useful.

<table>
<thead>
<tr>
<th>vertical line test</th>
</tr>
</thead>
<tbody>
<tr>
<td>a test for determining if a relation is a function</td>
</tr>
<tr>
<td>indicates that a relation is not a function when a vertical line intersects the graph of the relation at more than one point</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Functions</th>
<th>Non-Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relation A</td>
<td>Relation C</td>
</tr>
<tr>
<td>Equation (powers of $x$ and $y$)</td>
<td></td>
</tr>
<tr>
<td>Graph (linear, non-linear, or parabolic)</td>
<td></td>
</tr>
<tr>
<td>Table of values (number of $y$-values for each $x$-value)</td>
<td></td>
</tr>
<tr>
<td>Mapping diagram (number of arrows starting from each $x$-value)</td>
<td></td>
</tr>
</tbody>
</table>
**1.1 Identify Functions**

<table>
<thead>
<tr>
<th>Functions</th>
<th>Non-Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relation A</td>
<td>Relation C</td>
</tr>
<tr>
<td>Vertical line test</td>
<td>(number of points a vertical line intersects the graph)</td>
</tr>
</tbody>
</table>

2. Based on your results from parts A and B, explain the difference between a function and a non-function. Compare your answer with those of your classmates.

3. Describe a simple tool you used to test if a relation is a function.

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### Investigate B

**How is a function represented in function notation?**

A function \( f \) is like an input-output machine because it receives an input (the \( x \)-value), processes that input (by performing operations), and then produces the output (the \( y \)-value). When the input is \( x = 2 \), the result is written \( f(2) \). The statement \( f(2) = 5 \) means that the function \( f \) accepts 2 as the input, and \( y = 5 \) is the resulting output.

**A: Function Machine**

1. The numbers 0, 1, and 2 are used as the input for the function machine in the diagram.

![Function Machine Diagram]

The machine produces exactly one output, \( f(x) \), for each \( x \)-value.

2. Write the three results in the form \( f(a) = b \).

3. What are the values of \( f(3), \ldots, f(10) \)?

4. Describe in words what is happening to the input in order to produce the output.

5. **a)** If \( x \) is the input, what will be the output? Write it in the form \( f(x) = \) _.___

**b)** If \( n \) is the input, what will be the output? Write it in the form \( f(n) = \) _.___.
B: Area Function

1. Calculate the area of a circle of radius 1. Write it in the form \( A(1) = \) ___. 

2. If \( A(r) \) represents the area of a circle of radius \( r \), find \( A(3) \).

3. Describe in words how the number 3 is used to find the area.

4. If the radius is \( b \), what will be the area of the circle? Write it in the form \( A(b) = \) ___.

5. Express the formula for the area of a circle in the form \( A(r) = \) ___.

The letter \( f \) is often used to name a function; \( f(x) \) denotes the output of a function \( f \) for an input \( x \). In general, \( f(x) \) denotes the \( y \)-value of a function, so \( y = f(x) \). For example, the relation \( y = 3x - 4 \), which is a function, can be written as \( f(x) = 3x - 4 \). Other letters can be used, such as \( A \) for area and \( h \) for height, to model specific quantities. For example, the function \( h = 9.8t - 4.9t^2 \) that models the height \( h \), in metres, of a ball \( t \) seconds after being thrown may be written as \( h(t) = 9.8t - 4.9t^2 \).

Example: Identify Functions

For each relation, determine if it is a function. How do you know?

a) 

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

b) 

| \( y \) | 2 | 2 |
|---|---|
| \( x \) | 1 | 0 |

SOLUTION

a) **Method 1: Use the Properties of a Function**

In the mapping diagram, each value of \( x \) is mapped onto exactly one value of \( y \), so there is only one arrow starting from each value of \( x \). The relation is a function.
**Method 2: Graph the Data**

The points represented by the mapping diagram are (1, 6), (2, 6), (3, 5), (4, 4), and (5, 4).

Plot these points on a coordinate grid.

Since there are no two points lying on the same vertical line, the relation is a function.

**b) Draw a vertical line through the graph.**

Since a vertical line intersects the graph at more than one point, the relation is not a function.

**c) In this table of values, each value of \( t \) maps onto exactly one value of \( h \). The relation is a function.**

<table>
<thead>
<tr>
<th>( t )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
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<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

A vertical line test helps to determine if a relation is a function. However, a horizontal line test does not provide the same information.

**Making Connections**

Payroll software programs are designed to assist companies in managing the payroll of a large number of employees. Payroll software programs are like functions. They accept the input of data such as time and attendance of employees, perform calculations, and then produce the output—how much each employee is paid.
Key Concepts

- A relation is an identified pattern, or relationship, between two variables.
- Relations can be represented in a variety of ways: equations, tables, graphs, and mapping diagrams.
- A function is a relation between two variables such as \( x \) and \( y \) in which every value of \( x \) is mapped onto exactly one value of \( y \).
- Vertical line test: If a vertical line intersects the graph of a relation at more than one point, then the relation is not a function.
- \( f(x) = 2x + 1 \) is an example of a function written in function notation. It represents the same function as \( y = 2x + 1 \).

Communicate Your Understanding

C1 Explain the vertical line test: Why is a relation not a function if a vertical line intersects its graph at more than one point?

C2 True or False? All lines are functions. If it is true, explain. If it is false, give an example of a line that is not a function.

C3 Consider this spreadsheet.
Is the formula in cell B2 a function? Justify your answer.

Practise

For help with questions 1 to 4, refer to Example 1.

1. Determine if each relation is a function.
   a) \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & 0 \\
   -1 & 1 \\
   0 & 2 \\
   1 & 3 \\
   2 & 4 \\
   \end{array}
   \]
   b) \[
   \begin{array}{c|c}
   \text{Perimeter} & \text{Area} \\
   \hline
   5 & 2 \\
   10 & 3 \\
   15 & 9 \\
   10 & 6 \\
   5 & 4 \\
   \end{array}
   \]
   c) \[
   \begin{array}{c|c}
   r & c \\
   \hline
   0 & 0 \\
   1 & 6.28 \\
   2 & 12.56 \\
   3 & 18.84 \\
   4 & 25.12 \\
   \end{array}
   \]
   d) \[
   \begin{array}{c|c}
   x & y \\
   \hline
   0 & 0 \\
   1 & 2 \text{ or } -2 \\
   2 & 4 \text{ or } -4 \\
   3 & 6 \text{ or } -6 \\
   4 & 8 \text{ or } -8 \\
   \end{array}
   \]

2. Refer to question 1. For those relations that are not functions, explain why.
3. Determine if each relation is a function.

   a) ![Graph A]

   b) ![Graph B]

   c) ![Graph C]

4. Refer to question 3. For those relations that are not functions, explain why.

5. A relation can be expressed as a set of ordered pairs. Determine if each set of ordered pairs is a function. How do you know?
   a) {(1, 1), (2, 4), (3, 9), (4, 16)}
   b) {(-2, 0), (0, 2), (0, -2), (2, 0)}
   c) {(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)}

6. Refer to part c) of question 5.
   a) Plot the points on a coordinate grid.
   b) Explain how plotting the points helps determine if the relation is a function.

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**Connect and Apply**

7. Is the relation \(3x + 4y = 12\) a function? Give reasons for your answer.

8. Evaluate, given \(f(x) = 6x - 9\).
   a) \(f(4)\)
   b) \(f(-5)\)
   c) \(f\left(\frac{1}{2}\right)\)

9. Evaluate, given \(g(x) = x^2 + 3x + 2\).
   a) \(g(-2)\)
   b) \(g(0)\)
   c) \(g(3.14)\)

10. The height \(h\), in metres, of a ball \(t\) seconds after being thrown is modelled by the function \(h(t) = -4.9t^2 + 9.8t + 1\). A graph of Height versus Time shows the path of the ball.
    a) Describe in words the meaning of \(h(2)\) in this context.
    b) What is the height of the ball at the instant it is thrown?
    c) Describe how you can estimate when the ball will land. Give more than one method, if possible.
        Estimate the answer.

11. The graph represents a function \(f(x)\).

   ![Graph D]

   Use the graph to evaluate.
   a) \(f(-1)\)
   b) \(f(0)\)
   c) \(f(2)\)
12. Choose three professional sports teams and list two players on each team. Use the Internet or a newspaper to find teams and players, if necessary.

a) Use a mapping diagram to map teams to players.

b) Does your mapping diagram represent a function? Explain.

c) Will the mapping diagram represent a function if you reverse the arrows? Justify your answer.

13. Create two sets of data in which one represents a function and the other does not. Draw a mapping diagram for each set of data.

14. a) Explain how you can tell from each equation that the relation is not a function.

i) \(2x - y^2 = 1\)

ii) \(x^2 - y^2 = 0\)

iii) \(y^2 + 1 = x\)

iv) \(x^2 + y^2 = 1\)

b) Write the equations for two relations that are not functions.

15. Chapter Problem Research results for a concert show that 10 000 people will attend the concert if the price is $20 per ticket. For each dollar increase in price, 1000 fewer people will attend. 

Revenue = (number of people)\((\text{ticket price})\)

a) What will happen if the price is decreased by $1? Explain your thinking.

b) Create a table with three columns: Ticket Price, Number of People, and Total Revenue. Complete the table for ticket prices of $5, $10, $15, $20, $25, and $30.

c) If \(R(p)\) represents the total revenue for a ticket price of \(p\) dollars, find \(R(22)\).

16. Think of a function as a machine that will perform operations on whatever input it receives. Evaluate, given \(g(x) = 2x - 3\) and \(f(x) = x^2\).

a) \(g(f(2))\)

b) \(f(f(3))\)

c) \(g(g(4))\)

d) \(f(g(5))\)

17. If \(g(x)\) represents a linear function, and \(g(0) = 3\) and \(g(1) = 5\), then find an expression for \(g(x)\).

18. The graph represents the function \(g(x) = ax^2\).

a) Find the value of \(a\).

b) Find the value of \(g(5)\).

c) Find the value of \(g(g(2))\).

19. The equation for a relation is \(4x^2 + 9y^2 = 36\).

a) Is the relation a function? Give reasons for your answer.

b) Use Technology Explain how you can graph the relation, using a graphing calculator and using pencil and paper.
A dog run is built by enclosing a rectangular area along the side of a building with fencing material. What will then be the relationship between the width and the length of the run for a specified amount of fencing? If you know the width, what will you know about the length and the area?

Investigate A

How do you find the domain and range of a relation?

The diagram shows a dog run constructed using 40 m of fencing.

1. Create a two-column table of values for Width and Length.

2. For each value of width, describe in words how you calculated the corresponding length.

3. Write an equation expressing length in terms of width.

4. Graph Length versus Width. Is the relation a function? Give reasons for your answer.

5. Can the width be a negative value? Can it be over 25 m? Can it be a decimal? Explain why.

6. Describe in words all possible values for the width.

7. Describe in words all possible values for the length.

8. Add a third column to the table of values with the heading Area.

9. For each value of width, calculate the corresponding area.

10. Graph Area versus Width. Is the relation a function? Give reasons for your answer.

11. Describe in words all possible values for the area.
All possible widths of the dog run constitute the **domain** of the relation. For the width-length relation, all possible lengths of the dog run constitute the **range** of the relation. For the width-area relation, all possible areas of the dog run constitute the range of the relation.

Real-life factors can affect the domain and range of a relation. The 40 m of fencing restricts the possible values for width (independent variable) and length (dependent variable), as well as area. Both the domain and range of the relations are affected. If fence posts are set 2 m apart, the domain and range of the width-length relation will then be listed in increments of 2. The corresponding graph will be a finite number of points, as opposed to a continuous line or curve.

---

### Tools

- grid paper

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### Investigate B

**What real-life factors affect the domain and range of a relation?**

1. With a partner, decide if each item should be counted or measured. Sort the items into two columns.
   - angles in a triangle
   - arm span
   - books on a shelf
   - candies in a jar
   - CDs
   - distance travelled
   - gasoline
   - golf balls
   - mass of a block of cheese
   - price per kilogram
   - water bottles

<table>
<thead>
<tr>
<th>Count</th>
<th>Measure</th>
</tr>
</thead>
</table>

2. Discuss any item that has posed a problem or can go in either column. Explain why it posed a problem.

3. Add one item of your own to each column of the table. Can you think of any that can go in either column?

4. If gasoline costs $1.10 per litre, sketch a graph of Total Cost versus Number of Litres.

5. If golf balls cost $3 per package, sketch a graph of Total Cost versus Number of Packages.

6. Describe at least two ways the graphs in steps 4 and 5 are similar and one way they are different.

7. Explain how quantities that can be counted differ from those that can be measured in terms of the points on a graph.
The items or quantities involved in a relation will affect how the domain and range of the relation are expressed. The table shows several ways of expressing the domains and ranges of different graphs.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Domain (x-value)</th>
<th>Range (y-value)</th>
</tr>
</thead>
</table>
| ![Graph 1](image1) | **Integers**, or whole numbers, or natural numbers, from 2 to 5  
- \{2, 3, 4, 5\}  
- \{x \in \mathbb{I} \mid 2 \leq x \leq 5\} | **Integers**, or whole numbers, or natural numbers, from 1 to 4  
- \{1, 2, 3, 4\}  
- \{y \in \mathbb{I} \mid 1 \leq y \leq 4\} |
| ![Graph 2](image2) | Integers greater than or equal to -1  
- \{-1, 0, 1, 2, ...\}  
- \{x \in \mathbb{I} \mid x \geq -1\} | The integer 2  
- \{2\}  
- \{y \in \mathbb{I} \mid y = 2\} |
| ![Graph 3](image3) | **Real numbers** greater than or equal to -2 and less than or equal to 4  
- \{x \in \mathbb{R} \mid -2 \leq x \leq 4\}, or simply -2 \leq x \leq 4 | **Real numbers** greater than or equal to -3 and less than or equal to 2  
- \{y \in \mathbb{R} \mid -3 \leq y \leq 2\}, or simply -3 \leq y \leq 2 |
| ![Graph 4](image4) | Real numbers greater than or equal to 0  
- \{x \in \mathbb{R} \mid x \geq 0\}, or simply \(x \geq 0\) | Real numbers less than or equal to 2  
- \{y \in \mathbb{R} \mid y \leq 2\}, or simply \(y \leq 2\) |
| ![Graph 5](image5) | Real numbers  
- \{x \in \mathbb{R}\}, or simply \(x \in \mathbb{R}\) | Real numbers greater than or equal to 0  
- \{y \in \mathbb{R} \mid y \geq 0\}, or simply \(y \geq 0\) |

**Integers**
- all positive and negative whole numbers and zero  
- \(\mathbb{I}\) denotes the set of integers

**Literacy Connections**
- \(x \in \mathbb{I} \mid -2 \leq x \leq 5\) is an element of \(\mathbb{I}\) such that \(x\) is an element of the set of integers such that \(x\) is greater than or equal to -2 and less than or equal to 5.^

**Real Numbers**
- all integers, terminating decimals, repeating decimals, and non-terminating, non-repeating decimals  
- \(\mathbb{R}\) denotes the set of real numbers  
- \(\mathbb{R}\) produces a continuous line when graphed on a number line, as opposed to discrete points
Example 1  Domain and Range From Graphs

Write the domain and range of each relation.

a) \{(2, 8), (3, 7), (4, 6), (5, 5)\}

b) This is a curve on a graphing calculator screen. It does not show arrowheads at the open ends of the curve. However, it is understood that the curve continues at both ends, and that the scale is one unit per tick mark.

c) [Graph of a line showing x and y axes with intercepts and scale marks]

d) \(y = 1\)

SOLUTION

a) The domain and range can be expressed in a number of ways.

Method 1: List the Numbers

Domain: \{2, 3, 4, 5\}
Range: \{5, 6, 7, 8\}

Method: Write as a Set or as an Interval

Domain: \{x \in \mathbb{I} \mid 2 \leq x \leq 5\}
Range: \{y \in \mathbb{I} \mid 5 \leq y \leq 8\}

Method 3: Describe in Words

The domain is the set of integers from 2 to 5 inclusive. The range is the set of integers from 5 to 8 inclusive.

b) Domain: the set of real numbers, or \(\{x \mid x \in \mathbb{R}\}\), or \(\mathbb{R}\)
   Range: the set of real numbers greater than or equal to 0, or \(\{y \in \mathbb{R} \mid y \geq 0\}\), or \(y \geq 0\)

   c) Both the domain and range are the set of real numbers.
   Domain: \(\{x \mid x \in \mathbb{R}\}\), or \(\mathbb{R}\)
   Range: \(\{y \mid y \in \mathbb{R}\}\), or \(\mathbb{R}\)
d) The graph of \( y = 1 \) is a horizontal line 1 unit above the \( x \)-axis. Domain: the set of real numbers, or \( \{ x \mid x \in \mathbb{R} \} \), or \( \mathbb{R} \). Range: \( \{ y \mid y \in \mathbb{R} \mid y = 1 \} \), or \( \{ 1 \} \).

Example 2  \textbf{Real-Life Factors Affecting Domain and Range}

From the top of a 10-m cliff, a diver jumps 1 m into the air, does a front flip, falls, and hits the water 1.9 s after jumping.

a) Sketch a graph of Height versus Time for the relation that models the diver’s jump.

b) Write the domain of the relation.

c) Write the range of the relation.

\textbf{SOLUTION}

a) Let \( h \) represent the height of the diver, in metres, above the water \( t \) seconds after the jump. The graph of Height versus Time will be similar to this scatter plot from a graphing calculator.

b) The diver jumps when \( t = 0 \) and hits the water when \( t = 1.9 \).
   Domain: \( \{ t \in \mathbb{R} \mid 0 \leq t \leq 1.9 \} \)

c) The diver starts on the top of a 10-m cliff, jumps up 1 m to a maximum height of 11 m, and hits the water surface at 0 m.
   Range: \( \{ h \in \mathbb{R} \mid 0 \leq h \leq 11 \} \)

\textit{Assume that the minimum height is the water surface. Otherwise, if the diver went under water to a depth of 2 m, the range would be: \( \{ h \in \mathbb{R} \mid -2 \leq h \leq 11 \} \).}
Communicate Your Understanding

C1 A ball is dropped from a bridge at a height of 100 m above the ground and strikes the ground 4.5 s later. The ball’s height, \( h \), in metres, is graphed versus time, \( t \), in seconds. Which of the following best describes the range?

a) the set of real numbers

b) \( h > 0 \)

c) \( 0 \leq h \leq 100 \)

d) \( 0 < t < 4.5 \)

Explain why the others are not appropriate answers.

C2 Consider these two situations.

• A gas station sold between 30 and 60 litres of gas at $1.00 per litre.
• A variety store sold between 30 and 60 newspapers at $1.00 each.

The numbers involved in calculating the revenue from the sales are the same. How do the domain and range differ?

C3 True or False? For any line, the domain and range are always the set of real numbers. Justify your answer.

Practise

A

For help with questions 1 to 3, refer to Example 1.

1. Write the domain for each relation.

a) \( \{(0, 0), (1, 0), (2, 0), (3, 1), (4, 1), (5, 1)\} \)

b) \( f(x) = x^2 \)

c) \( x + y = 10 \)
2. For each relation in question 1, write the range.

3. For a circle of radius 5 and centred at the origin, express the domain and range in words and as intervals.

**Connect and Apply**

4. A rectangular field will be built using 100 m of fencing.
   a) Create a table of values using the length of the field as the independent variable.
   b) Draw a graph of Width versus Length.
   c) Which describes your graph, a series of dots or a line segment? Explain.
   d) Describe the domain and range in words.
   e) Should the domain and range include 0? Explain.
   f) How will the domain and range change if the length of the field must be at least 10 m and less than 20 m?

5. A ball is launched upward. Its height $h$, in metres, $t$ seconds after launching is modelled by the function $h = 19.6t - 4.9t^2$.
   a) Create a table of values relating $t$ and $h$.
   b) From the table, find the maximum height of the ball.
   c) From the table, determine how long the ball is in the air.
   d) Write the domain and range of the function.

6. The graph shows the cost of riding in a taxi. The pattern continues. The open dot at the right end of a line segment means that the point is not on the graph.

   a) What is the cost of a 4.3-km taxi ride?
   b) What does the open dot at the end of each line segment mean in this context?
   c) Will a 3-km taxi ride cost $4 or $5? Explain.
   d) Explain why the graph represents a function.
   e) Write the domain and range of the function.

7. Four squares are cut out from the corners of a square sheet of tin that measures 20 cm by 20 cm. Each cut-out square has a side length that is a whole number of centimetres. The rectangular sides are folded up to form a box with a square base and an open top.

   a) Create a table of values that relates the side length of the cut-out squares, in centimetres, and the volume of the box, in cubic centimetres, similar to this one.

<table>
<thead>
<tr>
<th>Side Length of Cut-Out Square (cm)</th>
<th>Volume of Box (cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

   b) What is the side length of the largest square that can be cut out from a corner? Explain your thinking.
   c) Write the domain of this relation.
8. **Chapter Problem** Research results for a concert show that 10 000 people will attend the concert if the price is $20 per ticket. For each dollar increase in price, 1000 fewer people will attend. Since Revenue = (number of people) (ticket price), the function that models the revenue of the concert is: \( R(x) = (10 000 - 1000x)(20 + x) \), where \( x \) represents the dollar increase in ticket price.

   a) Is it possible that \( x = -1 \)? What does it mean in this context?

   b) Is it possible that \( x = -5 \)? What must be considered when changing the price?

   c) What is the meaning of \( R(10) \) in this context? Evaluate \( R(10) \).

9. Sketch a graph for the function \( g(x) = 2x^2 + 9 \). Describe in words the domain and range of the graph. Explain your thinking.

10. The graph shows the cost of square carpets from a retail store.

   ![Image of the graph showing cost of carpet]

   a) Write the domain and range of the graph.

   b) In context of the question, describe any restriction on the range of the quadratic function that models the cost.

11. A relation that is a non-function can become a function by restricting the domain or range. The relation \( x^2 + y^2 = 25 \) for a circle with a radius of 5 and centred at the origin will be a function if the range is restricted to: \( 0 \leq y \leq 5 \).

   The domain of the function \( f(x) = 3x - 4 \) is restricted to \( 0 \leq x \leq 4 \). Find the range of this restricted function. Give more than one method, if possible. Explain each method.

**Extend**

12. For a certain function, the domain is \( \{1, 2, 3, 4, 5\} \) and the range is \( \{1, 2, 3, 4, 5\} \).

   a) Draw a mapping diagram to represent this function.

   b) Compare your diagram with that of another student. Is there more than one function possible? How many? Explain.

13. This is a graph for the relation \( xy = 1 \).

   ![Image of the graph showing xy = 1]

   a) Express \( y \) in terms of \( x \).

   b) Describe in words the domain and range of the relation.

   c) Express the domain and range in as many other ways as you can.

14. Think of a function as a machine that will perform operations on whatever input it receives. \( g(x) \) and \( f(x) \) are functions, given \( g(x) = 3x + 1 \) and \( f(x) = x^2 \).

   a) Write an expression for \( g(f(x)) \).

   b) Is the relation \( g(f(x)) \) a function? Explain.

   c) Write the domain and range of \( g(f(x)) \).
**Analyse Quadratic Functions**

Quadratic functions can be used to model the shape of the path of water from a fountain. Where else do you see this shape? Quadratic functions can also be used to model the area of a rectangular yard in relation to its length.

To study the increase in sales and profit in relation to the money spent on advertising, quadratic functions can help us answer the question: Will the profit continue to rise if the amount of advertising keeps increasing?

**Investigate A**

How do you use a graphing calculator to study a quadratic function that models profit?

Each year, a sporting goods store has a budget for advertising to generate more sales and thus profit. Analysis has found that the profit can be modelled by the function $P(x) = -2x^2 + 20x + 50$, where $x$ is the advertising budget and $P(x)$ is the profit, both in thousands of dollars.

1. Follow these steps to create a graph for the function.
   - Press $\text{Y}=$ and enter the equation $y = -2x^2 + 20x + 50$ for $Y1$.
   - Press $\text{2nd}$ [TABLE] and use the values to adjust the window settings by pressing $\text{WINDOW}$.
   - Press $\text{GRAPH}$.

2. What amount of advertising will produce the maximum profit? What is the maximum profit?

**Tools**

- graphing calculator

**Technology Tip**

To clear all equations, press $\text{Y}=$. To turn off all STAT PLOTS before you graph a function, press $\text{2nd}$ [STATPLOT]. Select 4:PlotsOff.
3. Write the domain and range in the context of the problem.

4. Press \[2\text{nd} \]\[TABLE\] and examine the maximum value.
   What will be the profit for $1000 more in advertising?
   for $1000 less?
   How do these values compare with each other?

5. Repeat step 4 for $2000 more in advertising, and then $2000
   less. What do you notice about the profit?

6. What property about this graph is suggested by your response to
   steps 4 and 5? List other properties of the graph.

7. Copy the table of values from the calculator for
   \(x = 0, 1, 2, \ldots, 12\). Then, calculate the \textit{first differences} and
   \textit{second differences}.

8. What do you notice in the first and second differences? What is
   their significance?

The function \(P(x) = -2x^2 + 20x + 50\) is an example of a quadratic
function. The graph of a quadratic function is a \textit{parabola}. The \textit{vertex}
is the point on the parabola where the maximum or minimum value
occurs. The vertex always lies on the \textit{axis of symmetry}.

The simplest parabola is the graph of the function \(y = x^2\), or \(f(x) = x^2\). The
graphs of functions in the form \(f(x) = a(x - h)^2 + k\) are also parabolas, and
they will be studied in later sections.

**Investigate B**

\textbf{How do you use a graphing calculator to find the equation of a
quadratic function?}

A baseball player throws a ball into the air. The height of the ball is
recorded every second until it hits the ground. The table shows the
results.

1. Copy the table. Then, calculate the
   first and second differences.

2. What do you notice in the first and
   second differences?

3. Sketch a graph of Height versus
   Time. Label the vertex and axis of
   symmetry, and write the domain and
   range.

4. Describe how the direction of opening
   of the graph determines if there is a
   maximum or minimum value.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>35</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>75</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>
5. Use a graphing calculator. Follow these steps to plot the data on a graph.

- Press \( \text{Y}= \) and clear any equations.
- Press \( \text{STAT} \). Select 1:Edit. Enter the values from the table into L1 and L2.
- Press 2nd [STAT PLOT]. Select 1:Plot1. Set the plot to On.
- Press \( \downarrow \) ENTER to select scatter plot.
- Press \( \downarrow \) 2nd [L1] to specify XList:L1.
- Press \( \downarrow \) 2nd [L2] to specify YList:L2.
- Press \( \downarrow \) ENTER to select the + mark for a data point.
- Press ZOOM. Select 9:ZoomStat to see the plot.

Follow these steps to find the equation of the quadratic function that models the data.

- Press \( \text{STAT} \) \( \blacktriangleright \). Select 5:QuadReg.
- Press 2nd [L1] 2nd [L2] \( \blacktriangleright \).
- Press VARS \( \blacktriangleright \). Select 1:Function.
- Press ENTER to select 1:Y1.
- Press ENTER to show the values of \( a \), \( b \), and \( c \) for the quadratic function \( y = ax^2 + bx + c \).

6. Write the resulting equation in the form \( y = ax^2 + bx + c \). Substitute some values of \( x \) (time) into this equation and note the results. Do the results make sense? Explain.

7. Press GRAPH. Describe what you see. Does the curve seem to pass through the points?

8. Describe how this process might help when analysing quadratic functions and functions of other types.

The QuadReg operation on a graphing calculator finds the quadratic equation that best fits a set of data. The graph of this equation is the curve of best fit for the data.
Example 1  
**Maximum Area for a Fixed Perimeter**

A community centre is building a new fence to surround a play area. The plan is to divide the area into two parts for different age groups.

The diagram shows how the two rectangular areas will be built using 120 m of fencing. Find the relationship between the length and area using a table of values and a graph. Then, find the length that will produce the maximum area.

**SOLUTION**

Create a table of values with these three columns: Length, Width, and Area.

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Width (m)</th>
<th>Area (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>33.3</td>
<td>333</td>
</tr>
<tr>
<td>20</td>
<td>26.6</td>
<td>533</td>
</tr>
<tr>
<td>30</td>
<td>20</td>
<td>600</td>
</tr>
<tr>
<td>40</td>
<td>13.3</td>
<td>533</td>
</tr>
<tr>
<td>50</td>
<td>6.6</td>
<td>333</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

When the width is 0 m, the length, 60 m, is the maximum. So, the domain (length) must be between 0 m and 60 m. From the diagram, $2 \times \text{length} + 3 \times \text{width} = 120$.

So, \( \text{width} = \frac{120 - 2 \times \text{length}}{3} \)

Use a graphing calculator to draw a scatter plot of Area versus Length.

If the points are connected, they will form a parabola. The graph is symmetrical about the axis of symmetry, \( x = 30 \).

This line passes through the vertex at (30, 600).

The vertex is the minimum point on a parabola that opens upward or the maximum point on a parabola that opens downward.

The maximum area will be 600 m² for a length of 30 m and a width of 20 m.
Use First and Second Differences to Identify Quadratic Relations

Determine if the relation is quadratic. Use first differences, and second differences, if necessary.

a)

\[
\begin{array}{c|c}
 x & y \\
-3 & 9 \\
-2 & 4 \\
-1 & 1 \\
0 & 0 \\
1 & 1 \\
2 & 4 \\
3 & 9 \\
\end{array}
\]

b)

\[
\begin{array}{c|c}
 \text{Days} & \text{Height} \\
0 & 105 \\
1 & 108 \\
2 & 111 \\
3 & 114 \\
4 & 117 \\
5 & 120 \\
6 & 123 \\
\end{array}
\]

**SOLUTION**

a) Find the first differences of the data. Since the first differences form a linear pattern, find the second differences.

\[
\begin{array}{c|c|c|c}
 x & y & \text{First Differences} & \text{Second Differences} \\
-3 & 9 & 4 - 9 = -5 & -3 - (-5) = 2 \\
-2 & 4 & 1 - 4 = -3 & -1 - (-3) = 2 \\
-1 & 1 & 0 - 1 = -1 & 1 - (-1) = 2 \\
0 & 0 & 1 - 0 = 1 & 3 - 1 = 2 \\
1 & 1 & 4 - 1 = 3 & 5 - 3 = 2 \\
2 & 4 & 9 - 4 = 5 & \\
3 & 9 & & \\
\end{array}
\]

The second differences are constant, so the relation is quadratic.

b) Find the first differences of the data.

\[
\begin{array}{c|c|c|c}
 \text{Days} & \text{Height} & \text{First Differences} \\
0 & 105 & 108 - 105 = 3 \\
1 & 108 & 111 - 108 = 3 \\
2 & 111 & 114 - 111 = 3 \\
3 & 114 & 117 - 114 = 3 \\
4 & 117 & 120 - 117 = 3 \\
5 & 120 & 123 - 120 = 3 \\
6 & 123 & & \\
\end{array}
\]

Since the first differences are constant, the relation is linear. There is no need to find the second differences.
Key Concepts

- The graph of a quadratic function is a parabola.
- The second differences of a quadratic function are constant.
- The equation in the form \( y = ax^2 + bx + c \) for a quadratic function can be found by using the QuadReg operation on a graphing calculator.

Communicate Your Understanding

C1 Describe the key features of a parabola. Include a diagram.

C2 Describe how to use first and second differences to determine if a function is quadratic.

C3 In the context of real-life situations, what is significant about the vertex of a parabola?

Practise

A

For help with questions 1 and 2, refer to Example 2.

1. Determine if each equation represents a linear or quadratic function.
   a) \( y = 2x + 1 \)
   b) \( f(x) = x^2 + 9 \)
   c) \( h(t) = -4.9t^2 + 19.6t + 2 \)
   d) \( 3x + 4y = 12 \)
   e) \( y = 1 \)
   f) \( g(x) = 2(x - 3)^2 \)
   g) \( y = \frac{2}{3}x - 5 \)
   h) \( f(x) = -13 - 0.9x^2 \)

2. For each set of data, identify the relation as linear, quadratic, or neither. Calculate the first differences and second differences, if necessary.

   a) \[
   \begin{array}{c|c}
   x & y \\
   \hline
   0 & 1 \\
   1 & 3 \\
   2 & 6 \\
   3 & 10 \\
   4 & 15 \\
   5 & 21 \\
   \end{array}
   \]

   b) \[
   \begin{array}{c|c}
   \text{Year} & \text{Population (millions)} \\
   \hline
   1999 & 25.0 \\
   2000 & 26.5 \\
   2001 & 28.0 \\
   2002 & 29.5 \\
   2003 & 31.0 \\
   2004 & 32.5 \\
   \end{array}
   \]

   c) \[
   \begin{array}{c|c}
   T & H \\
   \hline
   0 & 0 \\
   1 & 40 \\
   2 & 60 \\
   3 & 60 \\
   4 & 40 \\
   \end{array}
   \]

   d) \[
   \begin{array}{c|c}
   \text{Time (h)} & \text{Bacteria (billions)} \\
   \hline
   0 & 1 \\
   1 & 2 \\
   2 & 4 \\
   3 & 8 \\
   4 & 16 \\
   5 & 32 \\
   \end{array}
   \]
Connect and Apply

3. The height \( h \), in metres, of a ball \( t \) seconds after being thrown from a certain height is modelled by the function shown.
\[ h(t) = -4.9(t - 3)^2 + 60 \]

a) Create a table of values for \( t = 0, 1, 2, \ldots, 6 \).

b) Calculate the first and second differences.

c) Explain how your results in part b) show that the function is quadratic.

d) Give another reason why you know that the function is quadratic.

4. Refer to question 3.

a) Use the table of values to draw a graph for the function.

b) Identify the axis of symmetry, direction of opening, the coordinates of the vertex, and the domain and range.

5. A large \( 3 \times 3 \times 3 \) cm cube is made of 27 small cubes, each measuring \( 1 \times 1 \times 1 \) cm. The exterior of the large cube is painted.

a) How many of the small cubes have exactly one face painted?

b) If a \( 4 \times 4 \times 4 \) cm cube is assembled and painted in the same way, how many small cubes will have exactly one face painted?

c) Repeat part b) for a \( 5 \times 5 \times 5 \) cm cube and a \( 6 \times 6 \times 6 \) cm cube.

5. A large \( 3 \times 3 \times 3 \) cm cube is made of 27 small cubes, each measuring \( 1 \times 1 \times 1 \) cm. The exterior of the large cube is painted.

a) How many of the small cubes have exactly one face painted?

b) If a \( 4 \times 4 \times 4 \) cm cube is assembled and painted in the same way, how many small cubes will have exactly one face painted?

c) Repeat part b) for a \( 5 \times 5 \times 5 \) cm cube and a \( 6 \times 6 \times 6 \) cm cube.

d) Copy and complete the table.

<table>
<thead>
<tr>
<th>Side Length of Large Cube (cm)</th>
<th>Small Cubes with One Face Painted</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

e) Sketch a graph of Number of Small Cubes With Exactly One Face Painted versus Side Length. Is the relation quadratic? Justify your answer.

6. Sheets of tin measuring 50 cm by 70 cm have squares cut out at the corners as shown.

Each cut-out square has a side length that is a whole number of units. The four rectangular sides will then be folded up to make a box with an open top. Advertisements will be put on the four sides, so the total area needs to be a maximum.

a) Let \( x \) represent the side length of each cut-out square. What are the possible values of \( x \)? Explain your thinking.

b) Create a table of values with the side length of a cut-out square, \( x \), in the first column and the total area of the four sides of the box in the second column.

c) Find the first differences, and second differences, if necessary, to determine if the relation between the side length of each cut-out square and the total area for advertising is quadratic.

d) Use the table of values to determine the side length of each cut-out square that will produce the maximum area for advertising.
7. In a round robin tournament, each team must play every other team once.
   a) Copy and complete the table.

<table>
<thead>
<tr>
<th>Teams</th>
<th>Games Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

   Games Required means the total number of games played by all the teams in the tournament.

   b) What type of relation is this? Give reasons for your answer.

   c) How many games will be required for ten teams?

   d) Use Technology Enter the values from the table into a graphing calculator. Use the QuadReg operation to find an equation for the relation.

   e) Use the equation to find the number of games required for 25 teams.

8. Chapter Problem Research results for a concert show that 10 000 people will attend the concert if the price is $20 per ticket. For each dollar increase in price, 1000 fewer people will attend. The function that models the revenue of the concert is:

   \[ R(x) = (10000 - 1000x)(20 + x) \]

   a) Expand the function. Is it quadratic?

   b) Use Technology Graph the function using a graphing calculator. Is the graph a parabola?

9. A large cube measures \( n \times n \times n \), where \( n \) is a whole number measure of length. Find an expression for the number of unit cubes with exactly one face painted.

10. A circle has 15 points marked on its circumference. How many line segments are required to connect each point to all of the others? Hint: How is this related to question 7?

11. Use Technology Refer to question 6 on the previous page.
   a) Enter the values for \( x \) and the total area of the four sides into a graphing calculator. Use the QuadReg operation to find an equation for the function.

   b) Copy the diagram in question 6. Determine the dimensions of the four rectangular sides. Use these dimensions to find an expression for the total area of the four sides.

   c) Compare your results from parts a) and b). Are they the same? Explain.

Career Connection

Bianca completed a three-year course in video game development at an Ontario college. She was trained in the principles and techniques needed in the expanding game industry. Bianca now specializes in writing computer programs for video games that have been designed by her co-workers. Her job requires her to think logically and to incorporate mathematical equations into her programming code. Equations not only make the games function properly, they also help the animations appear more realistic.
Stretches of Functions

The mass of the moon is much less than that of Earth, which results in a smaller force due to gravity. The Men’s Olympic record for high jump is 2.39 m. If the record holder were to perform a record high jump on the moon, would his record be higher than it is on Earth?

Investigate

What is the role of $a$ in the quadratic function $y = ax^2$?

Method 1: Use a Graphing Calculator

1. Press $\text{Y}=$ and enter the three equations $y = x^2$, $y = 3x^2$, and $y = 0.5x^2$ for Y1, Y2, and Y3.

2. a) Graph the three equations on the same set of axes. Press $\text{WINDOW}$ and adjust the settings to display all three graphs.

   ![Graphs of $y = x^2$, $y = 3x^2$, and $y = 0.5x^2$]

   b) Describe how the graphs of $y = 3x^2$ and $y = 0.5x^2$ compare to the graph of $y = x^2$.

3. Press $\text{2nd}$ [TABLE] and copy the values into a table similar to this one.

   \[
   \begin{array}{c|c|c|c}
   x & y = x^2 & y = 3x^2 & y = 0.5x^2 \\
   \hline
   -3 & 9 & & \\
   -2 & 4 & & \\
   -1 & 1 & & \\
   0 & 0 & & \\
   1 & 1 & & \\
   2 & 4 & & \\
   3 & 9 & & \\
   \end{array}
   \]
4. How do the y-values for \( y = 3x^2 \) and \( y = 0.5x^2 \) compare to those for \( y = x^2 \)?

5. Press \( \text{Y} = \) and enter the three equations \( y = x^2, y = -x^2, \) and \( y = -2x^2 \) for Y1, Y2, and Y3.

6. a) Graph the three equations on the same set of axes.

   Press \( \text{WINDOW} \) and adjust the settings to display all three graphs.

   ![Graph of y = x^2, y = -x^2, y = -2x^2]

   b) Describe how the graph of \( y = -x^2 \) compares to the graph of \( y = x^2 \).

   c) Describe how the graph of \( y = -2x^2 \) compares to the graph of \( y = -x^2 \).

7. Copy and complete the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x^2 )</th>
<th>( y = -x^2 )</th>
<th>( y = -2x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. How do the y-values for \( y = -x^2 \) and \( y = -2x^2 \) compare to those for \( y = x^2 \)?

9. Describe the role of \( a \) in the quadratic function \( y = ax^2 \). Address in your response when \( a > 1, 0 < a < 1, \) and \( a < 0 \).

**Method 2: Use the Slider in Fathom™**

1. Go to [www.mcgrawhill.ca/functionsapplications11](http://www.mcgrawhill.ca/functionsapplications11) and follow the links to 1.4. Download the Fathom™ file for 1.4 Investigation. Open the sketch Vertical Stretch.

2. You will see two parabolas on the same set of axes: the blue one for \( y = x^2 \) and the red one for \( y = ax^2 \). You will also see a slider that you can use to change the value of \( a \). As you use your mouse to move the slider back and forth, the red parabola changes in shape.
3. Move the slider so that $a$ is greater than 1.

4. Describe how the red parabola compares to the blue one representing $y = x^2$.

5. Move the slider so that $a$ is between 0 and 1.

6. Compare and contrast the red and blue parabolas.

7. Move the slider past 0 so that $a$ is negative.

8. When $a$ becomes negative, what effect does $a$ have on the red parabola?

9. Slowly move the slider back and forth.

10. Describe the role of $a$ in the quadratic function $y = ax^2$.
    Address your response when $a > 1$, $0 < a < 1$, and $a < 0$.

Every point on the graph of $y = 3x^2$ is 3 times as far from the $x$-axis as the corresponding point on the graph of $y = x^2$. The graph of $y = ax^2$, where $a > 1$, is called a **vertical stretch** of the graph of $y = x^2$ by a factor of $a$.

**vertical stretch**
- a vertical stretch by a factor of $a$, where $a > 1$, means that every point on the graph is $a$ times as far from the $x$-axis as the corresponding point on the graph of $y = x^2$
- for each $x$-value, the $y$-value is multiplied by $a$
- the parabola appears narrower than the parabola for $y = x^2$ as the value of $a$ increases from 1
Every point on the graph of \( y = \frac{1}{2}x^2 \)
is half as far from the \( x \)-axis as the corresponding point on the graph of \( y = x^2 \). The graph of \( y = ax^2 \), where \( 0 < a < 1 \), is called a **vertical compression** of the graph of \( y = x^2 \) by a factor of \( a \).

The graph for \( y = -x^2 \) is a mirror image of the graph of \( y = x^2 \). The parabola opens downward instead of upward. The graph of \( y = ax^2 \), where \( a < 0 \), involves a reflection of the graph of \( y = x^2 \) in the \( x \)-axis.

Vertical stretches, compressions, and reflections are examples of **transformations**.

**Example 1  Graph a Vertical Stretch**

For each function, describe in words the transformation relative to \( y = x^2 \) and then sketch the graph.

a) \( y = 2x^2 \)

b) \( y = -3x^2 \)

**SOLUTION**

a) The graph of \( y = 2x^2 \) is a vertical stretch of the graph of \( y = x^2 \) by a factor of 2. Every point on the graph of \( y = 2x^2 \) is 2 times as far from the \( x \)-axis as the corresponding point on the graph of \( y = x^2 \).
b) The graph of \( y = -3x^2 \) includes a vertical stretch of the graph of \( y = x^2 \) by a factor of 3 and a reflection in the x-axis.

**Method 1: Stretch, then Reflect**

Multiply all y-coordinates by 3 to stretch the graph vertically by a factor of 3. Then, multiply all y-coordinates by -1 to reflect \( y = 3x^2 \) in the x-axis.

**Method 2: Reflect, then Stretch**

Multiply all y-coordinates by -1 to reflect \( y = x^2 \) in the x-axis. Then, multiply all y-coordinates of \( y = -x^2 \) by 3 to stretch the graph vertically by a factor of 3.
Example 2  Find the Stretch Factor

The point \((3, 12)\) is on the graph of the function \(y = ax^2\). Find the value of \(a\).

**Method 1: Use Algebra**

\[
y = ax^2
\]

\[
12 = a(3)^2
\]

\[
12 = 9a
\]

\[
a = \frac{12}{9}
\]

\[
a = \frac{4}{3}
\]

So, the value of \(a\) is \(\frac{4}{3}\) and the point \((3, 12)\) is on the graph of the function \(y = \frac{4}{3}x^2\).

**Method 2: Use Technology**

Use a slider in Fathom™. Move the slider to adjust the value of \(a\) until the graph of \(y = ax^2\) passes through the point \((3, 12)\).
The value of \(a\) is approximately 1.33.

Use a graphing calculator.
Press \(Y=\) and enter the two equations \(y = x^2\) and \(y = ax^2\) for \(Y1\) and \(Y2\).
Use systematic trial. Try values of \(a\) that are less than 1, such as \(\frac{2}{3}, \frac{1}{2}, \ldots\) or greater than 1, such as \(\frac{3}{2}, \frac{4}{3}, \ldots\)
For each value of \(a\), graph the two equations on the same set of axes.
Then, use the TRACE feature, or press \(2nd\) \([\text{CALC}]\), select \(1: \text{value}\), and enter the \(x\)-coordinate, to check if \(y = 12\) when \(x = 3\).

The value of \(a\) is \(\frac{4}{3}\).
Key Concepts

For a quadratic function of the form \( y = ax^2 \):

- The value of \( a \) determines if the graph represents a vertical stretch, a vertical compression, or a reflection in the \( x \)-axis relative to the graph of \( y = x^2 \).
  - If \( a > 1 \), the graph is a vertical stretch by a factor of \( a \).
  - If \( 0 < a < 1 \), the graph is a vertical compression by a factor of \( a \).
  - If \( a = -1 \), the graph is a reflection in the \( x \)-axis.
  - If \( -1 < a < 0 \), the graph is a vertical compression by a factor of \( -a \) and a reflection in the \( x \)-axis.
  - If \( a < -1 \), the graph is a vertical stretch by a factor of \( -a \) and a reflection in the \( x \)-axis.

- To graph a vertical stretch by a factor of 2, multiply each corresponding \( y \)-coordinate by 2.
- To graph a vertical compression by a factor of 0.5, multiply each corresponding \( y \)-coordinate by 0.5.
- To graph a reflection in the \( x \)-axis, multiply each corresponding \( y \)-coordinate by \(-1\).

Communicate Your Understanding

C1 A parabola has its vertex at the origin and passes through the point (4, 4). Explain how you know that this parabola represents a vertical compression of the graph of \( y = x^2 \) by a factor of 0.25.

C2 You are asked to graph the function \( y = -0.5x^2 \). Three methods are presented. Do all of them work? If so, which one will you use? Explain.

a) Compress the graph of \( y = x^2 \) vertically by a factor of 0.5. Then, reflect the resulting graph in the \( x \)-axis.

b) Reflect the graph of \( y = x^2 \) in the \( x \)-axis. Then, compress the resulting graph vertically by a factor of 0.5.

c) Use points on the graph of \( y = x^2 \) that are at the corners of grid squares, such as (1, 1) and (2, 4). Multiply each \( y \)-coordinate by \(-0.5\). Plot these points and join them with a smooth curve.
Practise

A

For help with questions 1 to 3, refer to Example 1.

1. Match each equation to the corresponding graph.
   a) \(y = 0.25x^2\)
   b) \(y = 3x^2\)
   c) \(y = -2x^2\)

2. Graph these three functions on the same set of axes. Label the graphs.
   a) \(f(x) = x^2\)
   b) \(g(x) = 4x^2\)
   c) \(h(x) = -0.5x^2\)

3. Draw a graph for \(y = x^2\). Then, draw the graph of each transformation relative to the graph of \(y = x^2\).
   a) a compression by a factor of 0.75
   b) a vertical stretch by a factor of 2.5
   c) a reflection in the \(x\)-axis and then a vertical stretch by a factor of 4
   d) a vertical stretch by a factor of 4 and then a reflection in the \(x\)-axis

   What do you notice about the graphs in parts c) and d)?

4. Write an equation for the graph that results from each transformation.
   a) The graph of \(f(x) = x^2\) is stretched vertically by a factor of 10.
   b) The graph of \(r(x) = x^2\) is compressed vertically by a factor of 0.25.
   c) The graph of \(t(x) = x^2\) is stretched vertically by a factor of 5 and reflected in the \(x\)-axis.

5. The point (5, 10) is on the graph of the function \(g(x) = ax^2\). Find the value of \(a\).

6. The graph of the function \(h(x) = ax^2\) is shown. What is the value of \(a\)?

Connect and Apply

B

7. a) How do you use the points on the graph of \(y = x^2\) to find the corresponding points on the graph of \(y = 4x^2\)?
   b) Draw the graphs of \(y = x^2\) and \(y = 4x^2\) on the same set of axes.

8. The point (3, 8) is on the graph of \(y = f(x)\).
   a) What is the \(y\)-coordinate of the point on the graph of \(y = 4f(x)\) if the \(x\)-coordinate is 3?
   b) Give reasons for your answer in part a).
9. On Earth, the vertical distance fallen by a free-falling object is represented by the function 
\[ d(t) = 4.9t^2, \] 
where \( d \) is the vertical distance fallen, in metres, and \( t \) is the time, in seconds.

a) How far does an object fall during the first second?

b) How long does it take an object to fall 100 m?

c) Nikayla drops a ball from the top of a building that is 45-m tall. How long does it take the ball to touch the ground?

10. On the moon, the acceleration due to gravity is less than that on Earth. On the moon, the vertical distance fallen by a free-falling object is represented by the function 
\[ d(t) = 0.81t^2, \] 
where \( d \) is the vertical distance fallen, in metres, and \( t \) is the time, in seconds.

a) How long does it take an object to fall 100 m on the moon?

b) What transformation on the graph of 
\[ d(t) = 0.81t^2 \] 
will give a graph that is the same as the graph of \( d(t) = 4.9t^2 \)?

11. Consider the bridge in the photograph. The shape of the bridge is close to a parabola.

a) Identify the coordinates of a point near the bottom of the parabolic bridge.

b) If the shape of the bridge is modelled by a function of the form \( b(x) = ax^2 \), use the point to find the value of \( a \).

12. The stopping distance of a car on dry asphalt can be modelled by the function 
\[ d(s) = 0.006s^2, \] 
where \( d(s) \) is the stopping distance, in metres, and \( s \) is the speed of the car, in kilometres per hour. The stopping distance for a car on wet asphalt can be modelled by the function 
\[ d(s) = 0.009s^2. \] The stopping distance for a car on ice can be modelled by the function 
\[ d(s) = 0.04s^2. \]

a) For each of the three surfaces, what is the stopping distance for a car travelling at 80 km/h?

b) Write a reasonable domain and range for each of the functions 
\[ d(s) = 0.006s^2, \]
\[ d(s) = 0.009s^2, \]
and 
\[ d(s) = 0.04s^2. \]

c) Compare the graphs of the three functions to the graph of \( y = x^2 \) by letting \( y \) represent \( d(s) \) and \( x \) represent \( s \).

13. An arrow is shot into the air. It lands at a point 25 m away from where it was shot.

a) If the velocity of a second arrow is increased, will the path of the second arrow represent a vertical stretch of the path of the first arrow? Use a diagram to help your reasoning.

b) Under what condition(s) will the path of the second arrow represent a vertical stretch of the path of the first arrow?

14. a) On the same set of axes, sketch the graphs of \( y = x \) and \( y = 2x \).

b) Compared to the graph of \( y = x \), would you say the graph of \( y = 2x \) is twice as tall or half as wide? Justify your answer.

c) Compared to the graph of \( y = x \), would you say the graph of \( y = 4x \) is taller or narrower? Justify your answer.
Translations of Functions

A javelin thrower launches a javelin along a parabolic path. The same thrower then runs 10 m past her original launch point before throwing a second javelin in exactly the same way. How are the equations representing the two throws alike? How are they different?

Investigate A

What is the role of \( h \) in the quadratic function \( f(x) = (x - h)^2 \)?

**Method 1: Use a Graphing Calculator**

1. Press \( \text{APPS} \) and select \text{Transfrm} to access the Transformation Graphing application.

2. Press \( \text{Y=} \) and enter \((X - A)^2\) for Y1.

3. Set \( A = 0 \). Press \( \text{WINDOW} \) to display the SETTINGS screen. The > | | symbol on the second line indicates the graph display type. Then, draw the graph.

**Technology Tip**

When you turn on the Transformation Graphing application, the opening page is displayed. When you press \( \text{ENTER} \), it appears that nothing is happening. However, when you press \( \text{Y=} \), you will notice that the line display symbol is different from the others that you are familiar with.

**Technology Tip**

To enter the variable \( A \), press \( \text{ALPHA} \) [MATH].
4. Press the → key. Describe what happens to the value of A and the graph as you do this. Repeat, using the ← key.

5. Without a graphing calculator, write the coordinates of the vertex of the parabola in the graph of \( g(x) = (x + 1)^2 \). Then, draw the graph. Use a graphing calculator to verify your result. Repeat for \( p(x) = (x - 3)^2 \).

6. Describe the role of \( h \) in the quadratic function \( f(x) = (x - h)^2 \).

7. Use the graphing calculator to graph \( f(x) = (x + 4)^2 \). Press [2nd] [TABLE] for a list of coordinates of the points near the vertex. What are the \( y \)-coordinates of the points 1 unit left/right of the vertex? 2 units left/right of the vertex?

8. Once you know the coordinates of the vertex, describe how you can find other points on the graph. Give reasons for your answer.

9. When you are finished, press APPS and select Transfrm. Select 1: Uninstall to shut down the Transfrm application.

**Method 2: Use the Slider in Fathom™**

1. Go to www.mcgrawhill.ca/functionsapplications11 and follow the links to 1.5. Download the Fathom™ file for 1.5 Investigation. Open the sketch Horizontal Translation.

2. You will see a parabola for the equation \( y = (x - h)^2 \). You will also see a slider that you can use to change the value of \( h \). Use your mouse to move the slider back and forth.
3. Write the equation when the value of $h$ is 3. Describe the graph. Does the shape of the graph change relative to the graph of $y = x^2$?

4. Without graphing, write the coordinates of the vertex of the parabola for $y = (x + 6)^2$. What is the value of $h$? Use the slider in Fathom™ to verify your answer.

5. What are the coordinates of another point on the graph? Explain your thinking.

6. Describe the role of $h$ in the quadratic function $f(x) = (x - h)^2$.

---

**Investigate B**

**What is the role of $k$ in the quadratic function $f(x) = x^2 + k$?**

**Method 1: Use a Graphing Calculator**

1. Use several different values of $k$, such as 0, 1, 2.5, 4, and $-3$. Graph each quadratic function in the form $f(x) = x^2 + k$ on a graphing calculator. What are the coordinates of the vertex of each parabola?

2. Using the graphs in step 1 as a guide, write the coordinates of the vertex of the parabola for $f(x) = x^2 + 5$. Use a graphing calculator to verify your answer.

3. Does the value of $k$ have any effect on the shape of the graph? Describe how you can find the coordinates of points on the graph of $f(x) = x^2 + 5$ other than the vertex.

**Method 2: Use the Slider in Fathom™**

1. Go to www.mcgrawhill.ca/functionsapplications11 and follow the links to 1.5. Download the Fathom™ file for 1.5 Investigation. Open the sketch Vertical Translation.

2. You will see a parabola for the equation $y = x^2 + k$. You will also see a slider that you can use to change the value of $k$. Use your mouse to move the slider back and forth.
3. Write the equation when the value of $k$ is 3. Describe the graph. Does the shape of the graph change relative to the graph of $y = x^2$?

4. Without graphing, write the coordinates of the vertex of the parabola for $y = x^2 - 10$. What is the value of $k$? Use the slider in *Fathom*™ to verify your answer.

5. What are the coordinates of another point on the graph? Explain your thinking.

6. Describe the role of $k$ in the quadratic function $y = x^2 + k$.

Compared to the graph of $f(x) = x^2$, the graph of $g(x) = (x - h)^2 + k$ represents a horizontal *translation* of $h$ units to the right and a vertical translation of $k$ units up. The coordinates of the vertex of the parabola for $g(x) = (x - h)^2 + k$ are $(h, k)$ and the shape of the graph is congruent to that of $f(x) = x^2$.

**Example** Graph Translations

For each of the following functions:

i) Describe in words the transformation relative to the graph of $f(x) = x^2$.

ii) Write the coordinates of the vertex.

iii) Sketch the graph. Label the vertex and one other point.

a) $f(x) = (x - 3)^2$

b) $g(x) = x^2 - 9$

c) $h(x) = (x + 2)^2 + 1$
SOLUTION

a) \( f(x) = (x - 3)^2 \)
   i) This represents a translation of 3 units to the right.
   ii) The vertex is at (3, 0).
   iii) The axis of symmetry divides a parabola into two congruent halves and intersects the parabola at the vertex.

\[ f(x) = (x - 3)^2 \]
\[ (1, 4) \]
\[ (5, 4) \]
\[ (0, 6) \]
\[ (6, 2) \]
\[ (2, 1) \]
\[ (4, 1) \]
\[ x = 3 \]

b) \( g(x) = x^2 - 9 \)
   i) This represents a translation of 9 units down.
   ii) The vertex is at (0, -9).
   iii) Since \( a = 1 \), the shape of the graph is not changed. From the vertex, move 1 unit to the left/right and 1 unit up; then, 1 unit to the left/right and 3 units up; then, 1 unit to the left/right and 5 units up, and so on to draw the graph.

\[ g(x) = x^2 - 9 \]
\[ (0, -9) \]
\[ (2, -5) \]

b) \( h(x) = (x + 2)^2 + 1 \)
   i) This represents a translation of 2 units to the left and 1 unit up.
   ii) The vertex is at (-2, 1).
   iii) Alternatively, move 1 unit left/right from the vertex and 1 unit up; then, 2 units left/right from the vertex and 4 units up, and so on to draw the same graph.

\[ h(x) = (x + 2)^2 + 1 \]
\[ (0, 5) \]
\[ (2, 1) \]
\[ (4, 5) \]
Key Concepts

Compared to the graph of the quadratic function $f(x) = x^2$:

- The graph of $f(x) = (x - h)^2$ represents a horizontal translation of $h$ units to the right.
  - For $h < 0$, a shift of $h$ units to the right means a shift of $-h$ units to the left.
- The graph of $f(x) = x^2 + k$ represents a vertical translation of $k$ units up.
  - For $k < 0$, a shift of $k$ units up means a shift of $-k$ units down.
- The coordinates of the vertex of the graph of $f(x) = (x - h)^2 + k$ are $(h, k)$.
  Similarly, the coordinates of the vertex of the graph of $f(x) = (x + h)^2 - k$ are $(-h, -k)$.
- Translations do not affect the shape of a graph.

Communicate Your Understanding

**C1** You know the coordinates of the vertex of a parabola that has been translated compared to the graph of $f(x) = x^2$. Describe how you use the vertex to find other points on the parabola.

**C2** Two points on the graph of the function $f(x) = (x - h)^2 + k$ are $(5, 4)$ and $(3, 4)$. Plot these points on a coordinate grid. Describe how you find the coordinates of the vertex.

Practise

For help with questions 1 and 3, refer to the Example.

1. Match each equation to the corresponding graph.
   a) $f(x) = (x - 4)^2$
   b) $g(x) = (x - 6)^2 - 2$
   c) $h(x) = (x + 6)^2 + 3$
   d) $k(x) = x^2 - 5$

2. Write an equation for the graph resulting from each transformation.
   a) The graph of $f(x) = x^2$ is translated 3 units to the right.
   b) The graph of $g(x) = x^2$ is translated 3 units up.
   c) The graph of $p(x) = x^2$ is translated 4 units to the left.
   d) The graph of $q(x) = x^2$ is translated 4 units down.

3. Write the coordinates of the vertex in each graph. Then, sketch the graph.
   a) $f(x) = x^2 - 4$
   b) $g(x) = (x + 3)^2$
   c) $h(x) = (x - 1)^2 + 6$
   d) $k(x) = (x + 5)^2 - 8$
Connect and Apply

4. Sketch the graphs for the functions \( y = x^2 - 1 \), \( y = x^2 - 4 \), and \( y = x^2 - 9 \) on the same set of axes.

   a) How are the \( x \)-intercepts alike for these three functions? Explain why this is the case.

   b) If the graphs are translated to the left or to the right, how will the equations of the functions change?

5. The \( x \)-intercepts of a parabola that opens upward are \( -6 \) and \( -2 \).

   a) What important feature of the parabola do you know from the \( x \)-intercepts?

   b) Find the equation of the parabola in the form \( p(x) = (x - h)^2 + k \).

6. Two identical dragsters are ready to start a race. One gives the other a 2-s head start. How are the position-time graphs for the two dragsters alike? How are they different?

7. The diagram shows a square-shaped lawn with a 6 m by 8 m pool built inside.

   a) Find an equation expressing the area of the lawn in terms of the side length of the square. Sketch a graph for the equation.

   b) Describe the transformation relative to a graph for the area of the lawn without the pool.

8. The graph of \( f(x) = x^2 \) is translated 2 units to the left and 4 units down.

   a) Write an equation for the graph resulting from the transformation.

   b) Sketch the graph.

   c) What are the coordinates of the vertex?

   d) What are the \( x \)-intercepts?

9. Relative to the graph of \( f(x) = x^2 \), the graph of \( g(x) = x^2 + 2 \) represents a translation of 2 units up, which is toward the positive direction. In Section 1.4, the graph of \( h(x) = 2x^2 \) represents a vertical stretch by a factor of 2, which is also toward the positive direction. However, the graph of \( f(x) = (x + 2)^2 \) represents a translation of 2 units to the left, which is toward the negative direction. How can you explain this apparent inconsistency?

Extend

10. The equation of a circle of radius 4 is given by \( x^2 + y^2 = 16 \). What will be the equation of the circle after a translation of 2 units to the right? How can you verify your answer?

11. The domain of the function \( f(x) = \sqrt{x} \) is \( \{ x \in \mathbb{R} \mid x \geq 0 \} \) and the range is \( \{ y \in \mathbb{R} \mid y \geq 0 \} \).

   a) Write the domain and range of the function \( g(x) = \sqrt{x - 3} + 2 \). Explain how you get your answer.

   b) How are the graphs of \( f(x) = \sqrt{x} \) and \( f(x) = -\sqrt{x} \) related? Graph the two equations on the same set of axes.

   c) Use the graphs in part b) to explain how \( f(x) = \sqrt{x} \) is related to a quadratic relation.
Two dragsters, Ben and Pat, are in a race. Ben sits at the starting line and then accelerates constantly until he reaches the finish line 400 m away.

The position-time graph shows how far Ben is from the starting line at different times.

Pat sits 100 m down the track and gives Ben a 2-s head start. How will the graph for Pat look when compared to that for Ben? Will Ben catch up with Pat?

### Investigate

What are the roles of \(a\), \(h\), and \(k\) in the quadratic function \(f(x) = a(x - h)^2 + k\)?

**Method 1: Use a Graphing Calculator**

1. Use a graphing calculator to graph the function \(y = (x - 2)^2 + 1\).
2. The vertex is located at (2, 1).
   Write the $y$-coordinates when $x = 1$ and $x = 3$.
   Verify by using one of the TRACE, CALCULATE, or TABLE features of the calculator.

3. Where are these two points relative to the vertex?
   Why does this make sense? Explain.

4. Repeat steps 2 and 3 for $x = 0$ and $x = 4$.

5. If you move 3 units to the left or to the right from the vertex, how many units do you expect to go up from the vertex to meet the graph?
   Explain your thinking.

6. On the same set of axes, graph the function $y = 3(x - 2)^2 + 1$.

7. How does this graph compare to that of $y = (x - 2)^2 + 1$?
   Does this graph have the same vertex? the same shape?

8. For the graph of $y = 3(x - 2)^2 + 1$, write the $y$-coordinates when $x = 1$ and $x = 3$.
   Verify your results using the TRACE feature on the graphing calculator.
   Why does this make sense?

9. For the graph of $y = 3(x - 2)^2 + 1$, if you move 3 units to the left or to the right from the vertex, how many units do you expect to go up from the vertex to meet the graph?
   Explain your thinking.
   How does this compare to your result in step 5?

10. For the graph of $y = -2(x + 5)^2 + 6$, describe in words where the vertex is and how you find the coordinates of two other points on the same graph.

**Method 2: Use the Slider in Fathom™**

1. Go to [www.mcgrawhill.ca/functionsapplications11](http://www.mcgrawhill.ca/functionsapplications11) and follow the links to 1.6. Download the Fathom™ file for 1.6 Investigation. Open the sketch Combining Transformations.

2. You will see a parabola for the equation $y = a(x - h)^2 + k$. You will also see three sliders that you can use to change the values of $a$, $h$, and $k$. Use your mouse to move each slider back and forth.
3. Experiment by moving the sliders and noting the effects on the parabola.

4. Move the sliders so the vertex is located at (2, 1).
   Which slider(s) do not affect the location of the vertex?
   Which slider(s) do not affect the shape of the graph?

5. Move the sliders to produce the graph of \( y = 3x^2 \).
   Identify a point on this graph other than the vertex.

6. Use the sliders to translate the graph of \( y = 3x^2 \) 4 units to the left and 2 units down.
   What are the new coordinates of the point you identified in step 5?

7. How can you use points on the graph of \( y = 3x^2 \) to plot points other than the vertex \((-4, -2)\) on the translated graph?

Example  Stretches and Translations

For the graph of the quadratic function \( f(x) = 3(x - 2)^2 - 9 \):

a) Describe in words the transformations relative to the graph of \( f(x) = x^2 \).

b) Write the coordinates of the vertex.

c) Write the equation of the axis of symmetry.

d) Sketch the graph. Label the vertex and two other points.

e) Write the domain and range of the function.

SOLUTION

a) Compared to the graph of \( f(x) = x^2 \), the graph of \( f(x) = 3(x - 2)^2 - 9 \) represents a vertical stretch by a factor of 3 and a translation of 2 units to the right and 9 units down.
b) The coordinates of the vertex are (2, −9).

c) The equation of the axis of symmetry is $x = 2$.

d) $f(x) = 3(x - 2)^2 - 9$

Since $a = 3$, from the vertex, move 1 unit to the left/right and $(3) \times 1$ units up; then, 1 unit to the left/right and $(3) \times 3$ units up; then, 1 unit to the left/right and $(3) \times 5$ units up, and so on to draw the graph. Alternatively, move 1 unit left/right from the vertex and $(3) \times 1$ units up; then, 2 units left/right from the vertex and $(3) \times 4$ units up, and so on to draw the same graph.

e) Domain: \{x \in \mathbb{R}\}  
Range: \{y \in \mathbb{R} \mid y \geq -9\}

For any parabola opening up or down, the domain remains as the set of real numbers. The range is determined by the $y$-coordinate of the vertex.

**Key Concepts**

- The graph of $y = a(x - h)^2 + k$ is congruent in shape to the graph of $y = ax^2$.

- Relative to the graph of $y = x^2$, the graph of $y = a(x - h)^2 + k$ represents a vertical stretch by a factor of $a$ and a translation of $h$ units to the right ($-h$ units to the left) and $k$ units up ($-k$ units down), resulting in a vertex of $(h, k)$.

- There can be more than one way to graph a function in the form $y = a(x - h)^2 + k$:  
  - Graph $y = ax^2$, and then translate all points on the graph $h$ units to the right ($-h$ units to the left) and $k$ units up ($-k$ units down).
  - Plot the vertex at $(h, k)$. From the vertex, draw a graph congruent in shape to the graph of $y = ax^2$. 


Communicate Your Understanding

C1 If the coordinates of the vertex of the graph of \( y = (x + 5)^2 - 9 \) are \((-5, -9)\), which ordered pair will give the coordinates of the vertex of the graph of \( y = -4(x + 5)^2 - 9 \)? Justify your answer.
   a) \((-20, -36)\)
   b) \((-5, 36)\)
   c) \((-5, -9)\)

C2 Explain how the graph of \( y = 2x^2 \) can be used to sketch the graph of \( y = 2(x + 3)^2 - 4 \).

C3 The graph of \( y = -4(x - 7)^2 + 11 \) is a result of several transformations on the graph of \( y = x^2 \). If you are to perform one transformation at a time, in which order will you perform the different transformations on the graph of \( y = x^2 \) to produce the graph of \( y = -4(x - 7)^2 + 11 \)? Explain your thinking.

Practise

A

1. Copy and complete the table.

<table>
<thead>
<tr>
<th>Equation of Parabola</th>
<th>Coordinates of Vertex</th>
<th>Equation of Axis of Symmetry</th>
<th>Direction of Opening</th>
<th>Range of Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = -5x^2 + 20 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 9(x - 5)^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 9(x - 5)^2 - 18 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = -2(x + 1)^2 + 32 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 0.5(x + 1)^2 - 3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For help with question 2, refer to the Example.

2. **Use Technology** Describe the graph of each function in terms of transformations on the graph of \( y = x^2 \). Then, sketch the graph. Clearly label the vertex, the axis of symmetry, and one other point. Check your result with a graphing calculator.
   a) \( y = 2(x + 3)^2 \)
   b) \( f(x) = -x^2 + 5 \)
   c) \( g(x) = 4(x + 2)^2 - 8 \)
   d) \( h(x) = -3(x - 1)^2 - 1 \)
3. The graph of \( f(x) = x^2 \) has been stretched vertically by a factor of 10 and translated 5 units to the right and 8 units down.
   a) Write the equation of the graph resulting from the transformations.
   b) Sketch the graph of \( f(x) = x^2 \) and its image after the transformations.

4. Write an equation of a parabola that satisfies each set of conditions.
   a) vertex \((4, -6)\)
      congruent in shape to the graph of \( y = 3x^2 \)
      range: \( \{y \in \mathbb{R} \mid y \leq -6\} \)
   b) vertex \((-2, 0)\)
      \(y\)-intercept: 4
   c) opens down
      congruent in shape to the graph of \( y = 2x^2 \)
      \(x\)-intercepts: 5 and 9

**Connect and Apply**

5. a) Describe how the graphs of the three functions are related.
   i) \( f(x) = (x + 2)^2 - 4 \)
   ii) \( g(x) = 2(x + 2)^2 - 4 \)
   iii) \( h(x) = -(x + 2)^2 - 4 \)
   b) Sketch the three graphs on the same set of axes to verify your answer in part a).

6. a) Write the coordinates of two points other than the vertex on the graph of \( f(x) = 2x^2 \).
   b) Explain how these points can help you draw the graph of \( g(x) = 2(x - 10)^2 - 32 \).
   c) Graph the function \( g(x) = 2(x - 10)^2 - 32 \). Label the vertex and the axis of symmetry and write the domain and range.

7. Consider the bridge in the photograph. The shape of the bridge is close to a parabola.
   a) Identify the coordinates of the vertex of the parabolic bridge.
   b) If the shape of the bridge is represented by \( b(x) = a(x - h)^2 + k \), use the vertex and other points on the parabola to find the value of \( a \).
   c) Describe how you can answer part b) using a different method.

8. The height, in metres, of a ball \( t \) seconds after being thrown is modelled by the function \( h(t) = -4.9(t - 2)^2 + 45 \).
   a) From what height is the ball thrown?
   b) What is the maximum height of the ball and when does this occur?
   c) Write the range of this function.
   d) **Use Technology** Use a graphing calculator to graph the function. Determine how long it takes the ball to land.
   e) Write the domain of the function.

9. **Chapter Problem** Research results for a concert show that 10 000 people will attend the concert if the price is $20 per ticket. For each dollar increase in price, 1000 fewer people will attend. The function that models the revenue of the concert is:
   \( R(x) = (10 000 - 1000x)(20 + x) \), where \( x \) represents the dollar increase in ticket price.
   a) Describe how the coordinates of the vertex of the graph can be determined with and without using technology.
   b) **Use Technology** Once the vertex has been determined, use it to express the revenue of the concert in the form \( R(x) = a(x - h)^2 + k \). Give more than one method, with and without technology.
10. On Earth, if an object is dropped from an initial height of \(h_0\) in metres, its approximate height above the ground \(h\), in metres, after \(t\) seconds is given by the function \(h(t) = -4.9t^2 + h_0\). On the moon, the object’s approximate height is given by the function \(h(t) = -0.8t^2 + h_0\). For an object dropped from an initial height of 20 m, the two functions will be \(h(t) = -4.9t^2 + 20\) and \(h(t) = -0.8t^2 + 20\).

a) Translate the graph of \(h(t) = -4.9t^2 + 20\) by 105 units up. Graph the function \(h(t) = -0.8t^2 + 20\) on the same set of axes.

b) Explain the meaning of the point that the resulting graph in part a) and the graph of \(h(t) = -0.8t^2 + 20\) have in common.

c) The approximate height of an object dropped on Jupiter is given by \(h(t) = -12.8t^2 + h_0\). For an initial height of 320 m, the function is \(h(t) = -12.8t^2 + 320\). Graph the two functions \(h(t) = -12.8t^2 + 320\) and \(h(t) = -0.8t^2 + 20\) on the same axes.

d) Describe how you can perform transformations on the graph of \(h(t) = -0.8t^2 + 20\) to result in the graph of \(h(t) = -12.8t^2 + 320\). Give reasons for your answer.

e) Explain the meaning of the point that the two graphs in part d) have in common.

f) Write the domain and range of each function in part d).

11. The coordinates of two points on a parabola are \((2, 20)\) and \((10, 20)\).

a) What important feature of the parabola do you know from these two points?

b) If a third point on the parabola is \((9, 34)\), find the equation of the parabola in the form \(f(x) = a(x - h)^2 + k\).

12. The bridge in the diagram is in the shape of a parabola. It is 10 m wide and the highest point of the bridge is 6 m above ground. A transport truck is 4.5 m tall and 3 m wide. The road is centred on the axis of symmetry and the transport truck must stay on its own side of the road. Will the truck fit under the bridge? Explain.

13. Use Technology Draw a picture, or design, using only portions of parabolas. The Geometer’s SketchPad® is one graphing software that can be used to generate a picture of this type. On a second copy of your picture, show the equation for each portion of a parabola in the first picture. Describe how you apply transformations to parts of the parabola for \(y = x^2\) to obtain each equation.
1. **Identify Functions, pages 6–14**

1. Is each relation a function? How do you know?
   a) 
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>
   b) \( y = x^2 - 4 \)
   c)

2. Evaluate, given \( f(x) = x^2 - 10x + 25 \).  
   a) \( f(3) \)  
   b) \( f(-5) \)  
   c) \( f\left(\frac{1}{2}\right) \)

3. The height \( h \), in metres, of a ball \( t \) seconds after being thrown is modelled by the function \( h(t) = -4.9t^2 + 20t \).
   a) Describe in words the meaning of \( h(2) \) in this context.
   b) What is the height of the ball 3 s after it is thrown?

4. **Domain and Range, pages 15–22**

4. Describe in words the domain and range of \( y = 2x + 1 \). Explain your thinking.

5. A parabola opens down and its vertex is located at \((-5, 10)\). Write the domain and range.

6. A catering company charges $50 plus $10 per person for a dinner with a maximum of 20 people. The catering cost can be modelled by the function \( C(x) = 10x + 50 \), where \( x \) represents the number of people. Write the domain and range of this function.

7. **Analyse Quadratic Functions, pages 23–30**

7. Does each relation represent a quadratic function? If not, explain why.
   a) \( h(t) = -2(t - 3)^2 + 10 \)
   b) \( 2x + y = 11 \)
   c) \( x^2 = 1 \)

8. For each set of data, identify the relation as linear or quadratic. Calculate the first differences, and second differences, if necessary.
   a) 
<table>
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<th>y</th>
</tr>
</thead>
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<td>2</td>
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</tr>
<tr>
<td>3</td>
<td>14</td>
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</tbody>
</table>
   b) 
<table>
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</tr>
<tr>
<td>2</td>
<td>19</td>
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<td>35</td>
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   c) 
<table>
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   d) 
<table>
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<th>Time (s)</th>
<th>Height (m)</th>
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<td>1</td>
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<td>45</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>
9. The diagram shows the first four terms of a pattern of cubes. Create a table of values and determine if the relation between Term Number and Number of Cubes is a quadratic function.

![Diagram showing the first four terms of a pattern of cubes]

10. For each function, describe the graph in terms of transformations on the graph of \( y = x^2 \). Then, sketch the graph. Label the vertex, axis of symmetry, and two other points.
   a) \( f(x) = -x^2 + 9 \)
   b) \( g(x) = 2(x - 5)^2 \)
   c) \( y = -4(x + 2)^2 + 8 \)

11. Write an equation for the parabola that satisfies each set of conditions.
   a) vertex at \((10, -5)\) congruent in shape to the graph of \( y = 3x^2 \) with no \( x \)-intercepts
   b) vertex at \((-3, 0)\) congruent in shape to the graph of \( y = 0.5x^2 \) range: \( y \in \mathbb{R} \mid y \leq 0 \)

12. Use Technology On the moon, the approximate height of an object above ground \( h \), in metres, \( t \) seconds after being dropped from a height of 50 m is given by the function \( h(t) = -0.8t^2 + 50 \).
   a) Graph the function using a graphing calculator.
   b) Use the TRACE feature to find the height of the object 4 s after being dropped.
   c) How long will it take the object to hit the ground?

### Chapter Problem Wrap-Up

The school’s Student Council has researched on the price of raffle tickets similar to the concert ticket scenario. The table shows the ticket price with an estimate of ticket sales and the corresponding revenue for several possible ticket prices.

<table>
<thead>
<tr>
<th>Ticket Price ($)</th>
<th>Estimated Sales</th>
<th>Revenue ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1200</td>
<td>3600</td>
</tr>
<tr>
<td>4</td>
<td>1100</td>
<td>4400</td>
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<tr>
<td>5</td>
<td>1000</td>
<td>5000</td>
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<td>600</td>
<td>5400</td>
</tr>
<tr>
<td>10</td>
<td>500</td>
<td>5000</td>
</tr>
</tbody>
</table>

a) Describe how first differences can be used to determine if the relation between ticket price and revenue is a quadratic function.

b) Use Technology How can you use a graphing calculator to find an equation of the form \( y = ax^2 + bx + c \) to model the revenue of the raffle ticket scenario?

c) How can you use the table of values to identify the vertex of the graph of the equation in part b)?

d) Describe how you use the vertex to express the same equation of the form \( y = a(x - h)^2 + k \).
For questions 1 to 5, choose the best answer.

1. Which relation is not a function?
   A \{ (1, 2), (2, 3), (3, 3), (4, 2) \}
   B \quad y = 2x - 9
   C \quad x^2 - 5 = y^2
   D \quad y = 1

2. Which set of numbers best represents the range of the parabola given by \( f(x) = -5(x - 9)^2 + 12 \)?
   A any real number greater than or equal to 12
   B \{ y \in \mathbb{R} \mid y \leq 12 \}
   C \{ 11, 10, 9, 8, \ldots \}
   D \{ y \in \mathbb{R} \mid y \leq 9 \}

3. Which statement is not true for the parabola given by \( h(t) = 3(t - 10)^2 - 50 \)?
   A Its vertex is located at (10, -50).
   B It opens up.
   C It passes through the point (5, 25).
   D The domain is any number greater than or equal to 10.

4. The point \((-2, -6)\) is on the graph of the function \( y = a(x + 1)^2 - 3 \). What is the value of \( a \)?
   A 1
   B 3
   C -1
   D -3

5. The graph of \( y = x^2 \) is compressed vertically by a factor of 0.5 and translated 2 units down. Which equation represents the resulting graph?
   A \quad y = 0.5x^2 + 2
   B \quad y = \frac{1}{2}x^2 - 2
   C \quad y = 2(x + 2)^2
   D \quad y = 2x^2 - 2

6. Determine if each relation is a function. Justify your answer.
   a)
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -3 & 12 \\
   -2 & 10 \\
   -1 & 8 \\
   0 & 6 \\
   1 & 4 \\
   2 & 2 \\
   3 & 0 \\
   \end{array}
   \]

7. Write the domain and range of each relation. Sketch a graph to help.
   a) \( y = 2x - 1 \)
   b) \( y = 2x^2 - 1 \)

8. The surface area, in square centimetres, of a cylinder with a height of 5 cm is given by the function \( S(r) = 2\pi r^2 + 10\pi r \).
   a) What is the meaning of \( S(5) \) in this context?
   b) Evaluate \( S(5) \).

9. Write an equation for the graph resulting from each transformation.
   a) The graph of \( f(x) = x^2 \) is translated 2 units left.
   b) The graph of \( h(t) = t^2 \) is translated 5 units up.
   c) The graph of \( A(r) = \pi r^2 \) is translated 4 units right.
   d) The graph of \( f(x) = 2x^2 \) is translated 3 units down.
10. Write the coordinates of the vertex in each graph.
   a) \( f(x) = x^2 + 2 \)
   b) \( g(x) = (x - 3)^2 + 1 \)
   c) \( h(x) = -(x + 1)^2 - 1 \)
   d) \( t(x) = 3(x + 1)^2 - 1 \)

11. Write an equation for the parabola that satisfies each set of conditions.
   a) vertex (1, 0)
      congruent in shape to the graph of \( y = x^2 \)
      range: \( \{ y \in \mathbb{R} \mid y \geq 0 \} \)
   b) vertex (0, 1)
      opens downward
      x-intercepts: \(-1\) and \(1\)

12. Describe the graph of each function in terms of transformations on the graph of \( y = x^2 \).
   a) \( y = x^2 + 7 \)
   b) \( y = (x - 1)^2 + 2 \)
   c) \( y = 5(x + 1)^2 - 4 \)
   d) \( y = -(x - 3)^2 + 5 \)

13. A parabola is modelled by the function \( g(x) = -(x - 4)^2 + 9 \).
   a) Sketch the parabola. Label the vertex, axis of symmetry, and two other points.
   b) Write the domain and range of the function.

14. The graph of the equation \( R(x) = (10 - x)(100 + 5x) \) is a parabola.
   a) Evaluate \( R(4) \).
   b) Describe two different methods you can use to show that the equation represents a quadratic function.
   c) Describe two different methods you can use to find the vertex.

15. The graph of the function \( f(x) = x^2 \) is stretched vertically and then translated 5 units to the left and 20 units down. If the \( y \)-intercept of the resulting graph is 30, find an equation for the function after these transformations.

16. A rectangular pen is to be built using 200 m of fencing. The diagram shows how the pen is divided into three sections.

- **Achievement Check**

- **16.** A rectangular pen is to be built using 200 m of fencing. The diagram shows how the pen is divided into three sections.

- **a)** Create a table of values with these three columns: Length, Width, and Area.
- **b)** Use length as the independent variable and area as the dependent variable. Write an equation representing the function that models the area. Write the domain and range.
- **c)** Determine the dimensions that will produce the maximum area.
- **d)** Write the vertex of the graph for the length-area function.
- **e)** Use other points on the same graph. Write an equation that models the area in the form \( y = a(x - h)^2 + k \).
- **f) Use Technology** Enter the values for length and area into L1 and L2 on a graphing calculator. Use the QuadReg operation to verify your result in part e).
How High Can My Plane Fly?

One of Jordan’s hobbies is to fly a battery-powered toy airplane. He is told that the height of his toy airplane can be modelled by the quadratic function \( h = -t^2 + 12t - 11 \), where \( h \) is the height, in metres, that the plane travels and \( t \) is the time of flight, in seconds.

Jordan wants to find the maximum height the airplane reached during its flight and the total time the plane was in the air.

1. Sketch a graph for the function \( h = -t^2 + 12t - 11 \) on a set of coordinate axis. Use values of \( h \) for \( y \) and values of \( t \) for \( x \).

2. Write the coordinates of the \( x \)-intercepts and the vertex from your graph. Is the vertex a maximum or a minimum?

3. Write the domain and range of your graph.

4. Write an equation for your graph in the form \( y = a(x - h)^2 + k \).
   Describe the graph in terms of transformations on the graph of \( y = x^2 \).

5. From your graph, where is the airplane at \( t = 0 \)? Does this make sense in context of the situation? Explain.

6. Do you think the airplane is flying between \( t = 0 \) and \( t = 1 \)? Explain your answer.
7. How would you restrict the domain and range of the function
   \[ h = -t^2 + 12t - 11 \] so that the function practically models the flight of the airplane at all times?

8. What is the maximum height that Jordan’s toy airplane reaches? How long was the plane in the air?